Math Mini-Test 1 Answer Explanations

1. **The correct answer is E.** To solve for *x*, subtract 5*x* and add 7 to both sides. The resulting equation is 2*x* = 9. Dividing both sides of the equation by 2 yields $x = \dfrac{9}{2}$.

2. **The correct answer is F.** Begin by using the distributive property to multiply *x* to (*y* − *w*) and *z*, giving *x*(*y* − *w*) + *xz*. Distribute *x* to (*y* − *w*) again to get *xy* − *xw* + *xz*.

3. **The correct answer is C.** To find the toxicity level, divide the number of milligrams of dissolved toxins per liter of water by the maximum safe amount of dissolved toxins in milligrams per liter of water, or 0.86 ÷ 1.04. Approximate the fraction as a decimal, 0.8269, then convert to a percent, 82.69%, and round it to 83%.

4. **The correct answer is G.** To find the length needed to fence a rectangular pasture 250 meters by 300 meters, you need to find the perimeter. The formula for the perimeter of a rectangle is twice the sum of the length and width, or *P* = 2(*l* + *w*) = 2(250 + 300) = 2(550) = 1,100. It may help you to draw out the pasture and then solve.

5. **The correct answer is B.** Since the average for the six tests is 88, multiply 88 by 6 to find out the total points Michael needs to get an average of 88. 88 · 6 = 528. Find the composite of the scores Michael already received. 72 + 94 + 85 + 83 + 97 = 431. Then subtract the composite score Michael already has from the desired composite score to get the score that he needs on the next exam. 528 − 431 = 97.

6. **The correct answer is J.** These 4 numbers will be an arithmetic sequence. In an arithmetic sequence, each pair of successive terms differs by the same amount. To find the difference, you can define *d* as the difference and let 19 be the first term and 55 be the fourth term. By definition, the second term is 19 + *d*, and the third term is 19 + *d* + *d*. The fourth term can also be written as 19 + *d* + *d* + *d*. Using that expression, you figure out that 55 = 19 + 3*d*, so *d* = 12. Then the second term is 19 + 12 = 31. The third term is 31 + 12 = 43. You may also solve this by plugging in the possible answer choices. Only choice J makes the difference between the consecutive numbers the same.

7. **The correct answer is E.** First, you need to figure out Mrs. Cook's pay per day. Divide her salary by the number of days she is paid. $23,125 ÷ 185 = $125 per day. When Mrs. Cook takes a day off without pay and the school pays a substitute teacher $90 instead, the school district saves the difference in these amounts, $125 − $90 = $35.

8. **The correct answer is J.** Probability is calculated as the number of *desired outcomes* divided by total number of *possible outcomes*. In this problem, there are 10 desired outcomes: drawing any of the 6 purple marbles or 4 blue marbles is desired: 6 + 4 = 10. The total number of possible outcomes is 6 + 4 + 10 = 20 marbles in the bag. So, the probability of the marble NOT being green is 10 ÷ 20 = $\dfrac{1}{2}$.

9. **The correct answer is B.** To find the number of distinct outfits that Zach can select from 3 pairs of shoes, 8 shirts, and 5 pairs of jeans, multiply the numbers of the 3 different clothing pieces together. Thus, there are (3)(8)(5) = 120 distinct outfits that Zach can select.

10. **The correct answer is K.** There are at least two ways to solve this. The first is to simply multiply the like terms together:

$4 \cdot 2 \cdot 3 = 24$ and $x^2 \cdot x^3 \cdot x = x^6 \cdot y \cdot y \cdot y^2 = y^4$, which gives us $24x^6y^4$.

If you are feeling a little uncomfortable with the problem, you could solve it the "long way." Remember that an exponent, such as the 6 on x^6, is just telling you how many times a number or variable is multiplied by itself.

For example, x^6 is the same as $x \cdot x \cdot x \cdot x \cdot x \cdot x$.

Or 5^6 is $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5$.

You can expand the expression given in the problem to get rid of the exponents:

$4 \cdot x \cdot x \cdot y \cdot 2 \cdot x \cdot x \cdot x \cdot y \cdot 3 \cdot x \cdot y \cdot y$

Then group the like terms:

$4 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

Then multiply it all together, combine like terms, and put your exponents back in place. Since there are 6 x variables, the exponent for the x variable is 6. Since there are 4 y variables, the exponent for the y variable is 4.

$4 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y = 24x^6y^4$

Math Mini-Test 2 Answer Explanations

11. **The correct answer is C.** The key to solving this problem is recognizing that there is a right triangle in the figure, with r as the hypotenuse. The bottom leg of the triangle is half the length of the chord. $20 \div 2 = 10$. Use the Pythagorean Theorem: $4^2 + 10^2 = r^2$. $r^2 = 116$. Use your calculator to find the square root of 116. $r = 10.770$. Round up to the nearest tenth to arrive at the answer: 10.8 inches.

12. **The correct answer is J.** To find out how high a 20-foot ladder 5 feet away from the base of a building can reach up the building, you can use the Pythagorean Theorem. Let the length of the ladder be the hypotenuse. Let one of the legs be the distance from the base of the building to the bottom of the ladder. Let the other leg be the height of the building up to where the ladder meets the building. This gives $20^2 = 5^2 + h^2$, where h is the height at which the ladder meets the building. Solving this equation for h and rounding yields $h = 19$. The height is about 19 feet.

$20^2 = 5^2 + h^2$

$400 = 25 + h^2$

$375 = h^2$

$h = 19.365$

Round down to 19.

It can help to draw this out. Be on the lookout for questions on the ACT where the test is describing a right angle being formed. Chances are you'll need to use the Pythagorean Theorem.

13. **The correct answer is B.** Points in Quadrant I have two positive coordinates. Points in Quadrant II have negative *x*-coordinates and positive *y*-coordinates. Points in Quadrant III have two negative coordinates. Points in Quadrant IV have positive *x*-coordinates and negative *y*-coordinates. Therefore, point *C* can only be in either Quadrant I or Quadrant III.

    If you aren't familiar with the rules governing this, you can always make up a point or two in each quadrant and see which quadrants match what the question is looking for.

14. **The correct answer is J.** The key to solving this problem is spotting that the point where these two lines intersect will have the same *x* and *y* values. You can find the *x*-coordinate by solving a system of equations. Since you know that $y = 2x - 1$, you can substitute $(2x - 1)$ for *y* in the second equation.

    $2x - 1 = x + 2$

    Now solve for *x*. Subtract *x* from both sides.

    $x - 1 = 2$

    Add 1 to both sides.

    $x = 3$

15. **The correct answer is D.** Because the circle is inscribed in the square, it touches the square at the midpoints of its 4 sides. For that reason, the length of the side of the square is the same as the circle's diameter. The diameter of a circle is twice the radius, so the diameter of this circle is $6 \cdot 2 = 12$. This is the same as the square's side length. The area of the square is, therefore, $12^2 = 144$ square feet.

16. **The correct answer is H.** To find the length of the diagonal of the rectangle, note that the diagonal of the rectangle and two of its sides form a right triangle. Therefore, you should use the Pythagorean Theorem. Where $12^2 + 16^2 = c^2$, *c* is the length of the diagonal. Solving this for *c* gives $c = 20$.

    The key to solving this problem is recognizing that you can use the Pythagorean Theorem. Drawing out the word problem can help you notice this, so be sure to make a habit of sketching a diagram for this sort of problem, especially if the way to solve it isn't immediately apparent.

17. **The correct answer is E.** The immediate answer to this question is that there is no number that is greater when you add 1 to it than when you add 8 to it, so it's the empty set.

    You can also solve this algebraically: Subtract 1 from both sides and *x* from both sides. You're left with $0 > 7$, which is never true. There is no solution to this problem and, therefore, it's the empty set.

    Note: Choice A is incorrect because *the set containing zero* doesn't mean *empty set* or *no solution*. That answer is saying that the solution is the number 0, which is incorrect.

18. **The correct answer is K.** To solve the equation $P = 2Q + 9$ for *Q*, subtract 9 from both sides of the equation to get $P - 9 = 2Q$. Then, divide both sides by 2 to get $Q = \dfrac{P - 9}{2}$. This is your answer.

    Sometimes this type of question with two variables provides you with two equations, which you can solve as a system of equations. When you feel like something is missing in the question, look down at your answer choices. The answers all have *P*s in them, so then you know you can solve for *Q* and leave the *P* in the expression.

19. **The correct answer is A.** There are several ways to solve this problem, but the easiest is to draw two circles that fit the description in the question. Draw one circle with radius 5 and another circle with radius 12.

Then find the circumference of each circle. Circumference = 2πr, so the circumference of the smaller circle is 2π(5) = 10π and the circumference of the larger circle is 2π(12) = 24π.

Then find the ratio of the two circumferences. 10π:24π. Simplify by dividing both sides of the ratio by π. 10:24. Simplify again by dividing both sides by 2. 5:12 is the correct answer. Choice D is not correct because the ratio is 10:24 (or 5:12) and not 10π:24.

Whenever the ACT describes the ratio between two shapes but does not give you specific dimensions, it almost always works to draw specific shapes by making up dimensions that fit the ratio in the problem and then working from there. It can make solving the problem much easier. As long as you take care to set up the shapes with the correct ratios, it will all come out correctly.

20. **The correct answer is F.** To find out how many of the 777 seniors in a certain high school are going to an art or design institute, first find out how many seniors are going to a trade school. This is given by $(777)(\frac{1}{3})$ = 259. The number of those seniors going to an art or design institute is $(259)(\frac{2}{7})$ = 74.

Math Mini-Test 3 Answer Explanations

21. **The correct answer is C.** Sin(L) = $\frac{5}{6}$ means that the ratio of the side length opposite of angle L and the hypotenuse side length is $\frac{5}{6}$. Set up the proportion to find the length of the opposite side. Use x to represent the unknown length MN.

$$\frac{5}{6} = \frac{x}{18}$$

Cross multiply.

$5 \cdot 18 = 6x$

$90 = 6x$

Divide both sides by 6.

$x = 15$

The abbreviation SOHCAHTOA can help you remember that sine of an angle is the ratio of the opposite length divided by the hypotenuse length in a right triangle (SOH).

22. The correct answer is K. The phrase *y in terms of x* means that you are to provide an equation with *y* all by itself, the variable *x* appearing somewhere on the other side of the equal sign. In other words, you're showing what *y* equals using *x*.

The first step is to get rid of the *t*. You can see in your answer choices that there is no *t* variable. Use the first equation to figure out what *t* equals in terms of *x*.

$x = 3t - 8$

Add 8 to both sides and divide by 3 to get *t* by itself.

$3t = x + 8$

$t = \dfrac{(x + 8)}{3}$

Substitute $\dfrac{(x + 8)}{3}$ in for *t* in the second equation.

$y = 4 + \dfrac{(x + 8)}{3}$

Add this together by finding the common denominator of 3, which gives $y = \dfrac{12}{3} + \dfrac{(x + 8)}{3} = \dfrac{(x + 20)}{3}$.

23. The correct answer is C. The easiest way to solve this is to draw a big octagon on your paper and draw all the possible diagonals. Methodically go from point to point in a clockwise direction. The first vertex has 5 diagonals coming from it (you can't connect it to the two vertices adjacent to it). The next vertex also has 5. The next vertex already has one of its diagonals counted, so that is 4. The next vertex has 3 new ones you can count. The next vertex has 2. The final vertex only has 1 new diagonal (all the rest are already connected to it). 5 + 5 + 4 + 3 + 2 + 1 = 20

24. The correct answer is K. Jennifer will count any candy other than chocolate, peppermint, licorice, and gum in the *other* category. The four main candies add up to 30 + 25 + 15 + 15 = 85%, leaving 15% for the *other* category. This means that 15% of the 360 degrees of the circle belong in the *other* sector. This is (0.15)(360) = 54 degrees.

The space around the center of a circle is divided into 360 pieces called degrees. When you're making graphs using circles, multiply the percentages by 360 to determine how much of the space in the circle is used by each category.

25. The correct answer is B. To find the number of program awards in afterschool programs, the number of participants in each program is multiplied by the ratio for that program, and then the four products are added.

(0.2)(30) + (0.3)(60) + (0.5)(40) + (0.4)(30)

6 + 18 + 20 + 12 = 56

26. **The correct answer is G.** The first trick to this problem is that even though the graphic looks like a parallelogram, it's just a three-dimensional representation of a rectangle. Therefore, the question is asking you to find the height of a prism with a length of 100 yards, a width of 68.5 yards, and a volume of 8,000 cubic yards (because that's how much sand will be piled on to this rectangle).

    Express this algebraically using the volume formula, $V = l \cdot w \cdot h$

    $8{,}000 = 100 \cdot 68.5h$

    $8{,}000 = 6{,}850h$

    Divide both sides by 6,850.

    $h = 1.168$

    This answer is between 1 and 2.

    Whenever the ACT gives you a volume, it's typically a clue that you'll be needing to use the volume formula to find a missing dimension (length, width, or height).

27. **The correct answer is B.** To find the distance between two points in the standard (x,y) coordinate plane, you can use the distance formula:

    $$d = \sqrt{\left(y_2 - y_1\right)^2 + \left(x_2 - x_1\right)^2}$$

    So the distance is:

    $$\sqrt{\left(6 - 2\right)^2 + \left(4 - 1\right)^2} = \sqrt{\left(16 + 9\right)} = \sqrt{25} = 5 \text{ units.}$$

    If you don't remember the distance formula, you can still solve this by drawing it out on the coordinate plane and using the Pythagorean Theorem. These points give you a right triangle with side lengths 3 and 4. Using the Pythagorean Theorem ($a^2 + b^2 = c^2$):

    $3^2 + 4^2 = c^2$

    $25 = c^2$

    Find the square root of both sides.

    $c = 5$

28. The correct answer is H. A trapezoid always has 2 parallel line segments. To find the measure of ∠WYX in the figure, you have to spot that line segments $\overline{VY}$ and $\overline{WX}$ are parallel and are connected by the line segment $\overline{WY}$. So, ∠XWY and ∠VYW are alternate interior angles. So ∠VYW is also 25 degrees. ∠VYZ is a straight angle, so by definition it is 180 degrees. Therefore, 25 + 110 + ∠WYX = 180, and ∠WYX = 45 degrees.

29. The correct answer is C.

$$\frac{i}{i-1} \cdot \frac{i+1}{i+1}$$

Multiply it out. $i(i+1) = i^2 + i$ is the numerator.

Use the FOIL method to determine the denominator. $(i-1)(i+1) = i^2 + i - i - 1 = i^2 - 1$

So the new fraction is:

$$\frac{i^2 + i}{i^2 - 1}$$

Substitute −1 for i^2.

$$\frac{i-1}{-2}$$

30. The correct answer is H. Plug in $(x + h)$ anywhere you see x in the function.

$(x + h)^2 + (x + h) + 4$

Use the FOIL method.

$(x + h)(x + h) + x + h + 4$

$x^2 + xh + hx + h^2 + x + h + 4$

Combine like terms.

$x^2 + 2xh + x + h^2 + h + 4$

Math Mini-Test 4 Answer Explanations

31. **The correct answer is C.** Draw a dotted line between M and O and place point X in the middle of that line; this is where the surveyor is located. Since line segment $\overline{LM}$ is 12 and line segment $\overline{QP}$ is 9, line segment $\overline{ON}$ is 3. Point X is halfway between O and N, therefore, point X is 9 + 1.5 = 10.5 blocks east of point L. You can also deduce that since line segment $\overline{LQ}$ is 14 and line segment $\overline{OP}$ is 5, line segment $\overline{MN}$ is 9. Point X is halfway between M and N, therefore, point X is 4.5 blocks north of point L.

If this starts to get confusing as you work it out, it can help to draw a big version of what the test provides you, and you can make a coordinate plane with little marks for each block inside the drawing.
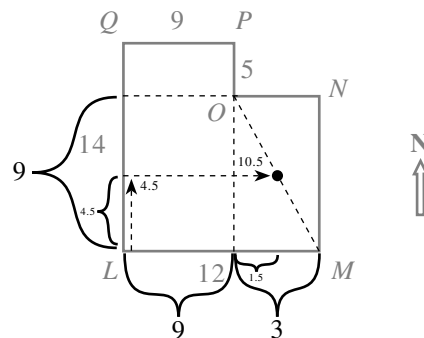
32. **The correct answer is G.** To start, we can find the equation that represents the diagonal line by using $y = mx + b$. We can use the identified points on the line to figure out that the slope is −1, and using that, we can determine that the line intersects the y-axis at (0, 1). That gives us an equation of $y = -x + 1$. In order to find the correct inequality that represents the graph, you can see that $y \leq -x + 1$ is satisfied for values of y under the sloped line. Also note that the inequality $x \geq 2$ is satisfied by all values of x to the right of the vertical line. This is the only pair of inequalities that can describe the graph.

When solving an inequality on a graph like this, always break it down to two different inequalities that explain part of what is bounding the shaded area.

You could also solve this by picking points inside and outside of the shaded area and seeing which answer choice works by plugging in those values to the inequalities given.

33. **The correct answer is C.** To find cos(θ), sketch a right triangle and use the given sin(θ) = $\frac{4}{5}$ to label what you know. Using SOHCAHTOA, we know that the side opposite θ is length 4, and the hypotenuse is length 5. Using the Pythagorean Theorem gives that the side adjacent to θ is length 3. This then gives that cos(θ) = $\frac{3}{5}$ for θ between 0 and $\frac{\pi}{2}$. However, because θ is between $\frac{\pi}{2}$ and π, cos(θ) = $-\frac{3}{5}$.

One way to describe angles is to use radians. There are 2π radians in a circle. If you are finding the cosine of an angle between $\frac{\pi}{2}$ and π then cosine will be negative. If you are having trouble remembering this rule, it can help to plug the different quantities into your calculator.

34. The correct answer is G. To find the coordinates of point P after it is reflected across the x-axis, notice that a reflection across the x-axis does not change the x-coordinate but does change the sign of the y-coordinate. Thus, the reflection of $P(a,b)$ across the x-axis is $P'(a,-b)$. It can help to draw this out to make sure you're not making a mistake.

The ' symbol means prime and is usually used to name the first new version or copy of something. For example, you might refer to Bob's reflection in the mirror as Bob' (said *Bob prime*).

35. The correct answer is E. Find $\cos\dfrac{\pi}{12}$ using $\cos(\alpha + \beta) = (\cos\ \alpha)(\cos\ \beta) + (\sin\ \alpha)(\sin\ \beta)$.

Given that $\dfrac{\pi}{12} = \dfrac{\pi}{3} + \dfrac{\pi}{4}$, substitute $\alpha = \dfrac{\pi}{3}$ and $\beta = \dfrac{\pi}{4}$ in the formula above.

$$\cos\frac{\pi}{12} = \cos(\frac{\pi}{3})\cos(\frac{\pi}{4}) + \sin(\frac{\pi}{3})\sin(\frac{\pi}{4})$$

Then use the table to convert the values.

$$(\frac{1}{2})(\frac{\sqrt{2}}{2}) + (\frac{\sqrt{3}}{2})(\frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

36. The correct answer is G. Express this word problem with algebraic equations. Express the first statement $y = 2x + 6$, where y is the larger number. Express the second statement as $2y + 4x = 70$. Solving this as a system of equations by substituting $2x + 6$ for y gives you $2(2x + 6) + 4x = 70$.

37. The correct answer is D. Although the shape is irregular, you know that the left sides add up to $12 + 5 = 17$. Therefore, even though you don't know what the right sides are, you know they also add up to 17. Likewise, the bottom side is 28; therefore, the top sides must add up to 28 as well. $17 + 17 + 28 + 28 = 90$ inches.

38. The correct answer is G. The question asks for a statement about the total number of dots in n rows. For example, looking at the figure, there are 9 *total* dots in the first 3 rows and 16 *total* dots in the first 4 rows. If you write next to the row marker the total number of dots for that row and all the rows before it, you'll see it goes 1, 4, 9, 16, 25, … This pattern is the row number squared, which is choice G.

You can also test each of the given equations for consistency with the dot pattern. After inspection, n^2 is the only equation that represents the dot pattern for all 5 displayed.

39. The correct answer is E. You must use the process of elimination. The standard equation of a parabola is given by $y = a(x - h)^2 + k$. The parabola in question opens downward, meaning that there must be a value with a negative sign. This eliminates choices A and B. Additionally, you are told it has a vertex not at the origin (0,0), meaning that (h,k) does not equal (0,0). This eliminates choice D. You know choice C is incorrect because a parabola has to have x^2, but choice C has y^2, leaving only $y = -4(x + 1)^2 - 3$ as a possibility.

40. The correct answer is H. To find the fraction of the total population living in Houston, divide the number of people living in Houston by the total population in all cities. This gives $\dfrac{2,000,000}{19,000,000}$, which simplifies to $\dfrac{2}{19}$ by dividing the numerator and denominator by 1,000,000.